A Schematic RH Proof for Dummies

How prime divisibility might lead to the critical line

A reader's guide to a *claimed*, not established, proof strategy

Status and scope (20 November 2026). Levent Alpöge has announced a purported AI-assisted proof of RH and Dirichlet GRH, accompanied by Lean files. Independent verification and a complete, human-readable mathematical exposition are not yet established. This document is a *schematic explanation of the reported strategy*, not a proof, an endorsement, or a reconstruction of the Lean development. In particular, the optimization formula below is illustrative, not a verified estimate from the proposed proof.

The whole argument in one minute

The hoped-for route has three stages:

Build arithmetic objects with controlled prime divisibility

⇓

Compare how divisible they must be with how big they can be

⇓

Obtain almost square-root estimates for weighted prime sums

⇓

Rule out zeros of relevant L-functions off the critical line

The first and last connections are familiar in spirit from elementary and analytic number theory. The difficult and unverified part is the middle: creating arithmetic objects with sufficiently strong *simultaneous* divisibility and size estimates.

1. What must eventually be bounded?

Define the von Mangoldt function and Chebyshev's function by

$$\Lambda(n) = \begin{cases} \log p, & n = p^k \ (k \geq 1), \\ 0, & \text{otherwise}, \end{cases} \qquad \psi(x) = \sum_{n \leq x} \Lambda(n).$$

The prime number theorem says $\psi(x) \sim x$. The Riemann hypothesis is equivalent to the much sharper estimate

$$\boxed{\psi(x) - x = O(x^{1/2}(\log x)^2).} \tag{1}$$

This illustrates the target: control the *fluctuation* of primes around their expected distribution with essentially square-root accuracy.

For Dirichlet GRH the analogous objects are character-twisted sums

$$\psi(x, \chi) = \sum_{n \leq x} \Lambda(n)\chi(n),$$

with the appropriate main term for the principal character. Often one uses *weighted or smoothed* variants because they fit the explicit formula better. The proposed argument is described as working with weighted von Mangoldt sums, rather than attacking $\pi(x)$ directly.

2. The elementary trick: divisibility puts a lower bound on size

Suppose a nonzero integer M is divisible by distinct prime powers p^{a_p}. Then

$$\prod_p p^{a_p} \le |M|, \qquad \text{so} \qquad \boxed{\sum_p a_p \log p \le \log |M|.}$$

This is an extremely simple bridge between *prime information* on the left and an ordinary *size bound* on the right.

A classical illustration is $N!$: its prime factorization and an estimate for $\log N!$ contain useful information about the primes. Chebyshev-type arguments exploit this general philosophy, although elementary versions fall far short of the error term in (1).

3. The amplification idea: make prime contributions count repeatedly

Imagine, just for intuition, that a construction made M divisible by p^{ma_p} instead of merely p^{a_p}. The same inequality becomes

$$\boxed{m \sum_p a_p \log p \le \log |M|.}$$

The multiplier m amplifies prime information. But increasing m may also make M much larger. **Amplification only helps if the extra divisibility is cheaper than the extra size.**

Here is the analogy with *Stepanov's method* for counting points over finite fields. A polynomial of degree D cannot have N different zeros each of multiplicity at least m unless

$$\boxed{mN \le D.}$$

High multiplicity makes every zero count multiple times, yielding a stronger bound. In Alpöge's description, *successive theta lifts* are intended to play a comparable amplifying role for arithmetic constructions. Theta lifting itself is an established technique; the asserted multiplicity-like effect in this setting is the proposed novelty.

Important warning about the analogy. A theta lift is not a derivative; m lifts do not automatically create zeros of order m, or an integer divisible by p^{ma_p}. The toy integer M is a teaching device. The actual reported constructions involve automorphic forms, lattices, and arithmetic maps. The exact lemma connecting repeated lifts to usable multiplicity/divisibility has to be stated and checked.

4. The balancing act: why a square-root number of repetitions?

The useful gain from repeating a construction typically competes with the cost of performing it. A toy error budget that captures the proposed intuition is

$$E(x, m) \;\approx\; \frac{x}{m} + m + \sqrt{x}. \tag{2}$$

The first term improves as we use more repetitions; the second grows with their cost. The last is a residual square-root term. By AM–GM,

$$\frac{x}{m} + m \geq 2\sqrt{x}, \qquad \text{with equality at } m = \sqrt{x}.$$

Thus the toy model selects $m \asymp \sqrt{x}$ and leaves an error of order $\sqrt{x}$. For $x = 10{,}000$:

m	x/m	m	$x/m + m + \sqrt{x}$
20	500	20	620
100	100	100	300
250	40	250	390

The announcement specifically describes a chain of about $\sqrt{X}$ theta lifts for a prime sum of length X. **Equation (2) explains why such a choice would be natural, but is not a claim that Alpöge has proved exactly this bound.**

5. Why does a prime-sum bound say anything about zeros?

The explicit formula connects weighted prime sums with the nontrivial zeros $\rho = \beta + i\gamma$ of $\zeta(s)$ or a Dirichlet L-function. Very schematically, once the expected main term is removed,

$$\text{prime-sum fluctuation} \quad \longleftrightarrow \quad \sum_{\rho} c_\rho x^\rho.$$

A zero with real part β can generate a contribution at scale x^β. If one proves the *appropriate uniform bound* essentially of size $x^{1/2+\varepsilon}$, analytic machinery can rule out zeros with $\beta > 1/2$; symmetry from the functional equation then yields the critical line.

The words *appropriate uniform bound* matter: a bound for an arbitrary selection or combination of sums does not by itself imply RH. The exact weights, character ranges, centering, and passage to the absence of off-line zeros must all be justified. This final analytic passage is logically separate from the hard arithmetic construction.

6. What is old, what is claimed new, and what is missing?

Part	**Status or role**
Explicit formula and RH equivalences	Classical analytic number theory.
Divisibility versus size	Classical idea, illustrated by elementary integers and factorials.
Stepanov analogy	An established function-field method used as motivation.
Repeated theta lifts	Established operation; its claimed *multiplicity-like amplification here* is unverified.
Strong simultaneous estimates	The decisive technical ingredient: usable divisibility *and* controlled size, at square-root precision.
Lean formalization	Reported by the claimant; reproduction, hypotheses, dependencies, and semantic fidelity require independent assessment.

7. What would an independent referee have to verify?

1. **The statement:** the final Lean declarations really assert ordinary RH and Dirichlet GRH, with the intended quantifiers and no additional hypotheses.
2. **The amplification:** successive theta lifts actually supply the specified arithmetic multiplicity or divisibility gains.
3. **The cost:** norms, denominators, dimensions, and other complexity terms obey bounds strong enough to preserve those gains.
4. **The analytic bridge:** the resulting prime-sum estimates are precisely strong and uniform enough to imply the required zero-free half-plane.
5. **Independence:** reproduce the proof-checking result and, ideally, scrutinize or rederive key components using a different formalization or mathematical route.

One-sentence summary. The proposed proof tries to turn a Chebyshev-like comparison between *prime divisibility* and *arithmetic size* into a square-root-accurate estimate for prime sums, by using repeated theta lifts as a substitute for Stepanov-style multiplicity; classical analytic number theory would then turn those estimates into RH/GRH.

A final perspective

Even if the full RH claim fails, a correct new lemma explaining how theta lifts amplify arithmetic divisibility—or sharp new estimates for the objects produced—could be valuable mathematics on its own. Conversely, an elegant narrative is not evidence that the crucial estimates actually hold.

Sources and further reading

1. Levent Alpöge, *Remark* (announcement dated 20 November 2026): `https://alpo.ge/remark.pdf`. This is the primary statement of the proposed Stepanov analogy and the roughly $\sqrt{X}$ theta-lift chain.
2. *State of Proof*, "A Claimed Proof of the Riemann Hypothesis" (20 November 2026): State of Proof: case record. Records the proposal and the need for independent examination.
3. H. M. Edwards, *Riemann's Zeta Function* (Dover). Background for explicit formulas and equivalent formulations of RH.
4. H. Davenport, *Multiplicative Number Theory*, 3rd ed., revised by H. L. Montgomery (Springer). Background for Dirichlet L-functions and prime sums.