MATH 214-2 - Fall 2002 - Practice Problems

1. Evaluate the integral $\displaystyle \int \frac{\cos x}{\sqrt{1 + \sin x}}\, dx$

2. Find the derivative of the function $\displaystyle F(x) = \int_{\sqrt{x}}^{x} \frac{e^t}{t}\, dt$

3. Evaluate the integral or show that it is divergent:

(a) $\displaystyle \int_0^\infty \frac{\ln x}{x^4}\, dx$ 　　　(b) $\displaystyle \int_0^1 \frac{1}{2 - 3x}\, dx$ 　　　(c) $\displaystyle \int_1^e \frac{dx}{x\sqrt{\ln x}}$

4. Approximate $\displaystyle \int_0^1 \frac{1}{1 + x}\, dx$ using Simpsom's Rule with 8 intervals. Do not perform the computations, just write the expression.

5. Find the area of the region bounded by the curves $y = e^x - 1$, $y = x^2 - x$ and $x = 1$.

6. The height of a monument is 20 m. A horizontal cross section at a distance x meters from the top is an equilateral triangle with side $x/4$ meters. Find the volume of the monument.

7. A 1600-lb elevator is suspended by a 200-ft cable that weights 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, at a distance of 30 ft?

8. Lengths of human pregnancies are normally distributed with mean 268 days and standard deviation 15 days. What percentage of pregnancies last between 250 days and 280 days?

9. Find the length of $y = \ln \sin x$ for $\frac{\pi}{6} \le x \le \frac{\pi}{3}$.

10. Solve the following initial value problem:

$$\begin{cases} \dfrac{dy}{dx} = 2x\,(y^2 + 1) \\ y(0) = 1 \end{cases}$$

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11. Assume the half-life of carbon 14 is 5700 years. A wooden statue is measured with 70% of the carbon-14. How old is the statue?

12. For each of the following series determine if it is convergent or divergent. If it is convergent find its sum:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{3n+2}}$
(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e^{2n}}$
(c) $\displaystyle\sum_{n=1}^{\infty} n \cdot \sin\left(\frac{1}{n}\right)$

13. Find the radius of convergence and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{5^n}$.

14. Evaluate $\displaystyle\int \frac{1}{1+x^5}\, dx$ as a power series.

15. Find the power series of $\dfrac{1+x}{1+x^2}$ about 0.

16. Find the Taylor series for $f(x) = \ln x$ at 1.

17. Use series to evaluate the following limit:
$$\lim_{x \to 0} \frac{1}{x^2} \left\{ \frac{\sin x}{x} - 1 \right\}$$

18. Solve the following initial value problem using power series:
$$\begin{cases} y'' = y \\ y(0) = 0 \\ y'(0) = 1 \end{cases}$$

19. The rate of spread of a rumor in a population is proportional to the product of the fraction y of the people who have heard the rumor and the fraction who have not heard the rumor. Write a differential equation that is satisfied by y and solve it.