CS 310 - Winter 2000 - Sample Midterm Exam

Last Name:	
First Name:	

1. (a) Prove the following by using truth tables:

$$\neg(p \to q) \Leftrightarrow (p \wedge \neg q)\,.$$

   (b) Write the following quantified statement in prenex normal form:

$$(\forall x\,(x > 0)) \to (\exists y\,(y < 0))\,.$$

2. Consider the following sets: $A = \{2x \mid (x \in \mathbb{Z}) \wedge (0 \leq x \leq 5)\}$, $B = \{x \in \mathbb{Z} \mid -5 \leq 2x \leq 5\}$. Find $A \cup B$, $A \cap B$, $A - B$, $B - A$, $A \triangle B$.

3. Prove that the following is an equivalence relation on $\mathbb{R}$:

$$x \,\mathcal{R}\, y \Leftrightarrow x - y \in \mathbb{Q}\,.$$

4. Let $f, g : \mathbb{R} \to \mathbb{R}$ be the functions $f(x) = x^2$, $g(x) = x + 3$. Find $g \circ f$, $f \circ g$, g^{-1}, $g^{-1} \circ f$, $f \circ g^{-1}$.

5. Find the properties (commutative, associative, existence of identity element, existence of inverse) verified by the following operation on $\mathbb{Z}$:

$$a * b = a + b + 1\,.$$

   Justify your answer.

6. Find how many 5-digits numbers (without leading zeroes) are not palindromes.